Diquark composites in the color superconducting phase of two flavor dense QCD

V.A. Miransky[a*], I.A. Shovkovy[bc†], L.C.R. Wijewardhana[b]

[a]Bogolyubov Institute for Theoretical Physics, 252143, Kiev, Ukraine

[b]Physics Department, University of Cincinnati, Cincinnati, OH 45221-0011, USA

[c]School of Physics and Astronomy, University of Minnesota, Minneapolis, MN 55455, USA

We study the Bethe-Salpeter equations for spin zero diquark composites in the color superconducting phase of $N_f = 2$ cold dense QCD. The explicit form of the spectrum of the diquarks, containing an infinite tower of narrow (at high density) resonances, is derived. It is argued that there are five pseudo-Nambu-Goldstone bosons (pseudoscalars) that remain almost massless at large chemical potential. These five pseudoscalars should play an important role in the infrared dynamics of $N_f = 2$ dense QCD.

In his studies Dmitrij Vasilievich Volkov was always led by the beauty within the problems he considered. One of his passions was the theory of spontaneous symmetry breaking, in particular, the dynamics of Nambu-Goldstone (NG) particles (both bosons and fermions), to which he contributed a great deal to our present understanding [1,2]. Therefore we think it is most appropriate to report in this volume a recent investigation of the dynamics of diquark composites (in particular, diquark NG bosons) in cold dense QCD with two fermion flavors.

Only a few years ago, not much was known about the properties of different phases in dense quark matter (see, however, Refs. [3,4]). The situation drastically changed after the ground breaking estimates of the color superconducting order parameter were obtained in Refs. [5,6]. Within the framework of a phenomenological model, it was shown that the order parameter could be as large as 100 MeV. Afterwards, the same estimates were also obtained within the microscopic theory, quantum chromodynamics [7–14]. The further progress in the field was mostly motivated by the hope that the color superconducting phase could be produced either in heavy ion experiment, or in the interior of neutron (or rather quark) stars.

Despite many advances [15–18] in study of the color superconducting phase of dense quark matter, the detailed spectrum of the diquark bound states (mesons) is still poorly known. In fact, most of the studies deal with the NG bosons of the three flavor QCD. At best, the indirect methods of Refs. [15–18] could probe the properties of the pseudo-NG bosons. It was argued in Ref. [19], however, that, because of long-range interactions mediated by the gluons of the magnetic type [7,8], the presence of an infinite tower of massive diquark states could be the key signature of the color superconducting phase of dense quark matter.

In this paper, we consider the problem of spin zero bound states in the two flavor color superconductor using the Bethe-Salpeter (BS) equations. We find that the spectrum contains five (nearly) massless states and an infinite tower of massive singlets with respect to the unbroken $SU(2)_c$ subgroup. Furthermore, in the hard dense loop improved ladder approximation, the following mass

*Present address: Department of Applied Mathematics, University of Western Ontario, London, Ontario N6A 5B7, Canada.

†On leave of absence from Bogolyubov Institute for Theoretical Physics, 252143, Kiev, Ukraine.

formula is derived for the singlets:

$$M_n^2 \simeq 4|\Delta|^2 \left(1 - \frac{\alpha_s^2 \kappa}{(2n+1)^4}\right), \quad n = 1, 2, \ldots, \tag{1}$$

where κ is a constant of order 1 (we find that $\kappa \simeq 0.27$), $|\Delta|$ is the dynamical Majorana mass of quarks in the color superconducting phase, and α_s is the value of the running coupling constant related to the scale of the chemical potential μ.

At large chemical potential, we also notice an approximate degeneracy between scalar and pseudoscalar channels. As a result of this parity doubling, the massive diquark states come in pairs. In addition, there also exist five massless scalars and five (nearly) massless pseudoscalars [a doublet, an antidoublet and a singlet under $SU(2)_c$]. While the scalars are removed from the spectrum of physical particles by the Higgs mechanism, the pseudoscalars remain in the spectrum, and they are the relevant degrees of freedom of the infrared dynamics. At high density, the massive and (nearly) massless states are narrow resonances.

In the case of two flavor dense QCD, the original gauge symmetry $SU(3)_c$ breaks down to the $SU(2)_c$ by Higgs mechanism. The flavor $SU(2)_L \times SU(2)_R$ group remains intact at the vacuum. The appropriate order parameter is an antitriplet in color and a singlet in flavor. Without loss of generality, we assume that the order parameter points in the third direction of the color space. In order to have a convenient description of the bound states at the true vacuum, we introduce the following Majorana spinors,

$$\Psi_a^i = \psi_a^i + \varepsilon_{3ab}\varepsilon^{ij}(\psi^C)_j^b, \qquad a = 1, 2, \tag{2}$$

$$\Phi_a^i = \phi_a^i - \varepsilon_{3ab}\varepsilon^{ij}(\phi^C)_j^b, \qquad a = 1, 2, \tag{3}$$

made of the Weyl spinors of the first two colors,

$$\psi_a^i = \mathcal{P}_+(\Psi_D)_a^i, \qquad (\psi^C)_j^b = \mathcal{P}_-(\Psi_D^C)_j^b, \tag{4}$$

$$\phi_a^i = \mathcal{P}_-(\Psi_D)_a^i, \qquad (\phi^C)_j^b = \mathcal{P}_+(\Psi_D^C)_j^b. \tag{5}$$

Here $i, j = 1, 2$ are flavor indices, $\mathcal{P}_\pm = (1 \pm \gamma^5)/2$ are the left- and right-handed projectors, Ψ_D is the Dirac spinor, and $\Psi_D^C = C\bar{\Psi}_D^T$ is its charge conjugate. Regarding the quark of the third color, we use the Weyl spinors, ψ^i and ϕ^i, for left and right components, respectively (notice that the color index is omitted).

The BS wave functions of the bound diquark states in the channels of interest are given by

$$(2\pi)^4\delta^4(p_+ - p_- - P)\boldsymbol{\chi}_a^{(\tilde{b})}(p, P) = \\ = \langle 0|T\Psi_a^i(p_+)\bar{\psi}_i(-p_-)|P; \tilde{b}\rangle_L, \tag{6}$$

$$(2\pi)^4\delta^4(p_+ - p_- - P)\boldsymbol{\lambda}_{(\tilde{a})}^{\ b}(p, P) = \\ = \langle 0|T\psi^i(p_+)\bar{\Psi}_i^b(-p_-)|P; \tilde{a}\rangle_L, \tag{7}$$

$$(2\pi)^4\delta^4(p_+ - p_- - P)\boldsymbol{\eta}(p, P) = \\ = \langle 0|T\Psi_a^i(p_+)\bar{\Psi}_i^a(-p_-)|P\rangle_L, \tag{8}$$

$$(2\pi)^4\delta^4(p_+ - p_- - P)\boldsymbol{\sigma}(p, P) = \\ = \langle 0|T\psi^i(p_+)\bar{\psi}_i(-p_-)|P\rangle_L, \tag{9}$$

where $p = (p_+ + p_-)/2$ and the quantities on the right hand side of these equations are defined as the Fourier transforms of the corresponding BS wave functions in the coordinate space. There are also the BS wave functions constructed out of the right handed fields Φ_a^i and ϕ^i. One might notice that there is another diquark channel, a triplet under $SU(2)_c$, that we do not consider here. The reason is that the repulsion dominates in such a channel, and no bound states are expected [the triplet comes from the $SU(3)_c$ sextet].

In order to derive the BS equations, we use the method developed in Ref. [20] for the case of zero chemical potential. To this end, we need to know the quark propagators and the quark-gluon interactions.

By introducing the multicomponent spinor that combines the Majorana spinors of the first two colors and the Weyl spinors of the third color, $\left(\Psi_b^j, \psi^j, \psi_j^C\right)^T$, we find that the inverse propagator takes the following block-diagonal form:

$$G_p^{-1} = \mathrm{diag}\left(S_p^{-1}\delta_a^{\ b}\delta^i_{\ j},\ s_p^{-1}\delta^i_{\ j},\ \bar{s}_p^{-1}\delta_i^{\ j}\right), \tag{10}$$

where, upon neglecting the wave functions renormalization of quarks [8–14],

$$S_p^{-1} = -i\left(\not{p} + \mu\gamma^0\gamma^5 + \Delta_p\mathcal{P}_- + \tilde{\Delta}_p\mathcal{P}_+\right), \tag{11}$$

$$s_p^{-1} = -i\left(\not{p} + \mu\gamma^0\right)\mathcal{P}_+, \tag{12}$$

$$\bar{s}_p^{-1} = -i\left(\not{p} - \mu\gamma^0\right)\mathcal{P}_-. \tag{13}$$

Here the notation, $\Delta_p = \Delta_p^+\Lambda_p^+ + \Delta_p^-\Lambda_p^-$, $\tilde{\Delta}_p =$

$\gamma^0\Delta_p^\dagger\gamma^0$, and $\Lambda_p^\pm = (1 \pm \vec{\alpha}\cdot\vec{p}/|p|)/2$ are the same as in Ref. [9].

The bare vertex, $\gamma^{A\mu}$, is also a 3×3 matrix,

$$\gamma^{A\mu} = \gamma^\mu \left(\begin{array}{ccc} \bar{\gamma}_{11}^A \delta^i_{\ j} & \bar{\gamma}_{12}^A \delta^i_{\ j} & \bar{\gamma}_{13}^A \varepsilon^{ij} \\ \bar{\gamma}_{21}^A \delta^i_{\ j} & \bar{\gamma}_{22}^A \delta^i_{\ j} & \bar{\gamma}_{23}^A \varepsilon^{ij} \\ \bar{\gamma}_{31}^A \varepsilon_{ij} & \bar{\gamma}_{32}^A \varepsilon_{ij} & \bar{\gamma}_{33}^A \delta_i^{\ j} \end{array} \right), \qquad (14)$$

with

$$\left(\bar{\gamma}_{11}^A\right)_a^{\ b} = T_a^{Ab} - 2\delta_8^A T_a^{8b}\mathcal{P}_-, \qquad (15)$$
$$\bar{\gamma}_{22}^A = T_3^{A3}\mathcal{P}_+, \qquad (16)$$
$$\bar{\gamma}_{33}^A = -T_3^{A3}\mathcal{P}_-, \qquad (17)$$
$$\left(\bar{\gamma}_{12}^A\right)_a = T_a^{A3}\mathcal{P}_+, \qquad (18)$$
$$\left(\bar{\gamma}_{13}^A\right)_a = -\varepsilon_{3ac}T_3^{Ac}\mathcal{P}_-, \qquad (19)$$
$$\left(\bar{\gamma}_{21}^A\right)^b = T_3^{Ab}\mathcal{P}_+, \qquad (20)$$
$$\left(\bar{\gamma}_{31}^A\right)^b = -T_c^{A3}\varepsilon^{3cb}\mathcal{P}_-, \qquad (21)$$
$$\left(\bar{\gamma}_{23}^A\right)_a = 0, \qquad (22)$$
$$\left(\bar{\gamma}_{32}^A\right)^b = 0, \qquad (23)$$

where T^A are the $SU(3)_c$ generators in the fundamental representation. By making use of this vertex and the propagator in Eq. (10), it is straightforward to derive the BS equations in the (hard dense loop improved) ladder approximation. The details of the derivation, as well as the explicit form of equations are given elsewhere [21]. Here we just note that the most transparent form of the equations appears for the amputated BS wave functions, defined by

$$\chi(p,P) = S^{-1}(p+\frac{P}{2})\boldsymbol{\chi}(p,P)s^{-1}(p-\frac{P}{2}), \qquad (24)$$
$$\lambda(p,P) = s^{-1}(p+\frac{P}{2})\boldsymbol{\lambda}(p,P)S^{-1}(p-\frac{P}{2}), \qquad (25)$$
$$\eta(p,P) = S^{-1}(p+\frac{P}{2})\boldsymbol{\eta}(p,P)S^{-1}(p-\frac{P}{2}), \qquad (26)$$
$$\sigma(p,P) = s^{-1}(p+\frac{P}{2})\boldsymbol{\sigma}(p,P)s^{-1}(p-\frac{P}{2}). \qquad (27)$$

In order to get a feeling of the problem at hand, let us briefly discuss the analysis of the BS equation for the χ-doublet. In general, the BS wave function contains eight different Dirac structures [22]. It is of great advantage to notice that only four of them survive in the center of mass frame, $P=(M_b,\vec{0})$,

$$\chi_a^{(\tilde{b})}(p,0) = \delta_a^{\ \tilde{b}}\hat{\chi}(p), \qquad (28)$$

where

$$\hat{\chi}(p) = \Big[\chi_1^-\Lambda_p^+ + (p_0 - \epsilon_p^- + \frac{M_b}{2})\chi_2^-\gamma^0\Lambda_p^+ + \chi_1^+\Lambda_p^- + (p_0 + \epsilon_p^+ + \frac{M_b}{2})\chi_2^+\gamma^0\Lambda_p^-\Big]\mathcal{P}_+, \qquad (29)$$

with $\epsilon_p^\pm = |\vec{p}| \pm \mu$ [the factors $(p_0 \pm \epsilon_p^\pm + M_b/2)$ are introduced here for convenience]. This is the most general structure that is allowed by the space-time symmetries of the model.

Now, in the particular case of the NG bosons, $M_b = 0$, we will show that the BS wave function is fixed by the Ward identities. Indeed, let us consider the following non-amputated vertex:

$$\boldsymbol{\Gamma}^{A,i}_{aj,\mu}(x,y) = \langle 0|Tj_\mu^A(0)\Psi_a^i(x)\bar{\psi}_j(y)|0\rangle, \qquad (30)$$

where, for our purposes, it is sufficient to consider $A=4,\ldots,8$ (that correspond to the five broken generators). In the (hard dense loop improved) ladder approximation, the vertex satisfies the following Ward identity [21]:

$$P^\mu\boldsymbol{\Gamma}^{A,i}_{aj,\mu}(k+P,k) = iT_a^{A3}\delta_j^i\left[s_k - S_{k+P}\right]\mathcal{P}_-. \qquad (31)$$

As in the case of the BS wave functions, it is more convenient to deal with the corresponding amputated quantity,

$$\Gamma^{A,i}_{aj,\mu}(k+P,k) = S^{-1}_{k+P}\boldsymbol{\Gamma}^{A,i}_{aj,\mu}(k+P,k)s_k^{-1}. \qquad (32)$$

This latter satisfies the following identity:

$$P^\mu\Gamma^{A,i}_{aj,\mu}(k+P,k) = iT_a^{A3}\delta_j^i\left[S^{-1}_{k+P} - s_k^{-1}\right]\mathcal{P}_+. \qquad (33)$$

By making use of the explicit form of the quark propagators in Eqs. (11) and (12), we could check that the right hand side of Eq. (33) is non-zero in the limit $P\to 0$. This is possible only if the vertex on the left hand side develops a pole as $P\to 0$. After a simple calculation, we obtain

$$\Gamma^{A,i}_{aj,\mu}(k+P,k)\Big|_{P\to 0} \simeq \frac{\tilde{P}^\mu}{P_\nu\tilde{P}^\nu}T_a^{A3}\delta_j^i\tilde{\Delta}_k\mathcal{P}_+, \qquad (34)$$

where, we introduced $\tilde{P}^\mu = (P_0, c_\chi^2\vec{P})$ with c_χ being the velocity of the NG boson in the χ-doublet channel.

By making use of the definition in Eqs. (6) and (24), it is also not difficult to show that the pole contribution to the vertex function (34) is directly

related to the BS wave function. By omitting the details,

$$\chi_a^{(\tilde{a})}(p,0) \equiv \delta_a^{(\tilde{a})}\chi(p,0) = \delta_a^{(\tilde{a})}\frac{\tilde{\Delta}_p}{F^{(\chi)}}\mathcal{P}_+, \quad (35)$$

where $F^{(\chi)}$ is the decay constant of the corresponding doublet whose formal definition is given by

$$\langle 0| \sum_{A=4}^{7} T_a^{A3} j_\mu^A(0)|P,\tilde{b}\rangle_L = i\delta_a^{\tilde{b}}\tilde{P}_\mu F^{(\chi)}. \quad (36)$$

By comparing the Dirac structures in Eqs. (29) and (35), we see that no components of the $\chi_2^\pm$ type appear in Eq. (35) which follows from the Ward identities. It was rewarding to establish that, in this approximation, the structure of the BS wave function required by the Ward identity is indeed a solution to the BS equation for the χ-doublet. A similar situation takes place for the η-singlet [23].

Now let us discuss the fate of the massless states that we obtain. Altogether, there are five scalars and five pseudoscalars (a doublet, an antidoublet and a singlet). Because of the Higgs mechanism, the scalars are removed from the spectrum. Nevertheless, these scalar bound states exist in the theory as "ghosts" [24], and one cannot get rid of them completely, unless a unitary gauge is found. In fact, these ghosts play a very important role in getting rid of unphysical poles from the on-shell scattering amplitudes [24].

As for the pseudoscalars, they remain in the spectrum as pseudo-NG bosons. In the (hard dense loop improved) ladder approximation, they look like NG bosons because the left and right sectors of quarks decouple. One could think of this as an effective enlargement of the original color symmetry from $SU(3)_c$ to an approximate $SU(3)_{c,L} \times SU(3)_{c,R}$. Then, since the approximate symmetry of the ground state is $SU(2)_{c,L} \times SU(2)_{c,R}$, five scalar NG bosons (which are removed by the Higgs mechanism) and five pseudoscalar NG bosons (which remain in the spectrum) should appear. Of course, in the full theory, the pseudoscalars are only pseudo-NG bosons. Indeed, they should get non-zero masses due to higher orders corrections that are beyond the improved ladder approximation [25]. Since the theory is weakly coupled at large chemical potential, it is natural to expect that the masses of the pseudo-NG bosons are small compared to the value of the dynamical quark mass.

We conclude our discussion of the massless diquarks by emphasizing that the low-energy dynamics of the two flavor QCD is dominated by massless quarks of the third color (which might eventually get a small mass too if another (non-scalar) condensate is generated [5,26]) and by the five pseudoscalars that remain almost massless in the dense quark matter. Of course, the gluons (glueballs) of the unbroken $SU(2)_c$ may also be of some relevance but we do not study this question here.

Now, let us consider massive diquarks. The structure of the BS equations becomes even more complicated in this case. In addition, one does not have a rigorous argument to neglect the component functions like $\chi_2^\pm$ in Eq. (29). In spite of this, we argue that all the approximations made before might still be reliable. Indeed, from the experience of solving the gap equation (which coincides with the BS equation for the massless states), we know that the most important region of momenta in the integral equation is $|\Delta| \ll p \ll \mu$. In this region, the kernel of the BS equations for massive states, $M_b \lesssim |\Delta|$, is almost the same. The deviations appear only in the infrared region where $p \lesssim |\Delta|$.

Therefore, in our analysis of the BS equations for massive states, we closely follow the approximation used for the massless diquarks. By assuming that the component functions depend only on the time component of the momentum (compare with the analysis of the gap equation in Refs. [8–14]), we arrive at the following equation for the BS wave function of the massive singlets:

$$\eta_1^-(p) = \frac{\alpha_s}{4\pi}\int_0^\Lambda dq K^{(\eta)}(q)\eta_1^-(q)\ln\frac{\Lambda}{|q-p|}, \quad (37)$$

where $\Lambda = (4\pi)^{3/2}\mu/\alpha_s^{5/2}$, and the kernel reads

$$K^{(\eta)}(q) = \frac{\sqrt{q^2+|\Delta|^2}}{q^2+|\Delta|^2-(M_\eta/2)^2}, \quad (38)$$

[$\eta_1^-(p)$ is a scalar function that appears in the decomposition of the BS wave function η over

the Dirac matrices (compare with Eq. (29)]. At this point it is appropriate to emphasize that the Meissner effect plays an important role in the analysis of the massive bound states. Indeed, our analysis shows that these massive states are quasiclassical in nature, i.e., their binding energy is small compared to the value of the gap [see Eq. (1)]. As a result, only the long range interaction mediated by the unscreened gluons of the unbroken $SU(2)_c$ is strong enough to produce these diquark states. We took this into account in Eq. (37). For completeness, we mention that the (nearly) massless (pseudo-) NG bosons are tightly bound, and the Meissner effect is not so important for their binding dynamics.

By approximating the kernel (38) in each of the following three regions: $0 < q < \sqrt{|\Delta|^2 - (M_\eta/2)^2}$, $\sqrt{|\Delta|^2 - (M_\eta/2)^2} < q < |\Delta|$ and $|\Delta| < q < \Lambda$, we could solve the BS equation (37) analytically. Then, by matching the logarithmic derivatives of the separate solutions, we obtain the spectrum of the massive diquarks. By omitting the details, it is presented in Eq. (1).

We would like to emphasize that for large μ the hard dense loop improved ladder approximation is reliable for the description of those bound states. The point is that a) the region of momenta primarily responsible for the formation of these composites is $E_{bind} \lesssim q \ll \mu$, where the binding energy $E_{bind} \sim \alpha_s^2 \Delta$, and b) $E_{bind} \to \infty$ as $\mu \to \infty$ [8–11]. Therefore the vacuum effects are higher order ones in α_s in that region. Because of that, the hard dense loop improved ladder approximation, in which the contribution of the vacuum effects to the running of the coupling constant is neglected and only the running due to the polarization effects provided by the quark matter (non-zero μ) is taken into account, is justifiable for large μ.

Now, let us consider the case of massive diquarks in the doublet channel. As is easy to check, the binding interaction in this channel is exclusively due to the five gluons affected by the Meissner effect. The approximate BS equation looks similar to Eq. (37), but with a different kernel and $|q-p|$ in the logarithm replaced by $|q-p| + c\Delta$ where $c = O(1)$ is a constant [9]. At high density when the coupling constant is weak, this equation does not allow a non-trivial solution for $M \neq 0$. From the physical viewpoint, this indicates that the heavy gluons, with $M_{gl} \sim (\alpha_s \mu^2 \Delta)^{1/3} \gg \Delta$, cannot provide a sufficiently strong attraction to form massive radial excitations of the NG and pseudo-NG bosons.

At the end, let us note that the massive diquark states may truly be just resonances in the full theory, since they could decay into the pseudo-NG bosons and/or gluons (glueballs) of the unbroken $SU(2)_c$. At high density, however, both the running coupling $\alpha_s(\mu)$ and the effective Yukawa coupling $g_Y = |\Delta|/F \sim |\Delta|/\mu$ [16–18,27]) are small, and, therefore, these massive resonances are narrow.

In conclusion, in this paper we studied the problem of diquark bound states in the color superconducting phase of $N_f = 2$ dense QCD. While the scalar NG bosons are ghosts in the theory, the pseudoscalar pseudo-NG bosons are physical particles that should play an important role in the infrared. We also obtained the spectrum of the massive narrow diquark resonances, whose existence would be a clear signature of the unscreened long range forces in dense QCD.

Acknowledgments. V.A.M. thanks V.P. Gusynin for useful discussions. The work of V.A.M. and I.A.S. was partly supported by the Grant-in-Aid of Japan Society for the Promotion of Science No. 11695030. The work of I.A.S. was supported by the U.S. Department of Energy Grants No. DE-FG02-84ER40153 and No. DE-FG02-87ER40328. The work of L.C.R.W. was supported by the U.S. Department of Energy Grant No. DE-FG02-84ER40153.

REFERENCES

1. D.V. Volkov, Preprint of the Institute for Theoretical Physics ITF-69-75, Kiev, 1969; Sov. J. Part. Nucl. **4** (1973) 1.
2. D.V. Volkov and V.P. Akulov, Phys. Lett. B **46** (1973) 109.
3. B.C. Barrois, Nucl. Phys. **B129** (1977) 390; S.C. Frautschi, in “Hadronic matter at extreme energy density”, edited by N. Cabibbo and L. Sertorio (Plenum Press, 1980).

4. D. Bailin and A. Love, Nucl. Phys. **B190** (1981) 175; *ibid.* **B205** (1982) 119; Phys. Rep. **107** (1984) 325.
5. M. Alford, K. Rajagopal and F. Wilczek, Phys. Lett. B **422** (1998) 247.
6. R. Rapp, T. Schaefer, E.V. Shuryak and M. Velkovsky, Phys. Rev. Lett. **81** (1998) 53.
7. R.D. Pisarski and D.H. Rischke, Phys. Rev. Lett. **83** (1999) 37.
8. D.T. Son, Phys. Rev. D **59** (1999) 094019.
9. D.K. Hong, V.A. Miransky, I.A. Shovkovy and L.C.R. Wijewardhana, Phys. Rev. D **61** (2000) 056001.
10. T. Schafer and F. Wilczek, Phys. Rev. D **60** (1999) 114033.
11. R.D. Pisarski and D.H. Rischke, Phys. Rev. D **61** (2000) 05150.
12. S.D.H. Hsu and M. Schwetz, Nucl. Phys. **B572** (2000) 211.
13. W.E. Brown, J.T. Liu and H.-C. Ren, Phys. Rev. D **61** (2000) 114012; *ibid.* D **62** (2000) 054016.
14. I.A. Shovkovy and L.C.R. Wijewardhana, Phys. Lett. B **470** (1999) 189; T. Schafer, Nucl. Phys. **B575** (2000) 269.
15. R. Casalbuoni and R. Gatto, Phys. Lett. B **464** (1999) 111; hep-ph/9911223; D.K. Hong, M. Rho, and I. Zahed, Phys. Lett. B **468** (1999) 261.
16. D.T. Son and M.A. Stephanov, Phys. Rev. D **61** (2000) 074012.
17. M. Rho, A. Wirzba and I. Zahed, Phys. Lett. B **473** (2000) 126; M. Rho, E. Shuryak, A. Wirzba and I. Zahed, Nucl. Phys. **A676** (2000) 273.
18. D.K. Hong, T. Lee and D.-P. Min, Phys. Lett. B **477** (2000) 137; C. Manuel and M.G.H. Tytgat, *ibid.* B **479** (2000) 190; K. Zarembo, Phys. Rev. D **62** (2000) 054003; S.R. Beane, P.F. Bedaque and M.J. Savage, Phys. Lett. B **483** (2000) 131.
19. V.A. Miransky, I.A. Shovkovy and L.C.R. Wijewardhana, Phys. Lett. B **468** (1999) 270.
20. P.I. Fomin, V.P. Gusynin, V.A. Miransky and Yu.A. Sitenko, Riv. Nuovo Cimento, **6** No. 5 (1983) 1.
21. V.A. Miransky, I.A. Shovkovy and L.C.R. Wijewardhana, Phys. Rev. D **62** (2000) 085025.
22. Note that in the case of zero chemical potential there are only four different Dirac structures, see Ref. [20].
23. In connection with the Ward identities, it is appropriate to mention here the complementary analysis in W.E. Brown, J.T. Liu and H.-C. Ren, Phys. Rev. D **62** (2000) 054013. The authors of this paper consider the contribution to the Ward identity that is directly related to the quark wave function renormalization.
24. R. Jackiw and K. Johnson, Phys. Rev. D **8** (1973) 2386; J.M. Cornwall and R.E. Norton, *ibid.* D **8** (1973) 3338.
25. An example of such higher order corrections is the box diagram in the BS kernel with two intermediate gluons. We would like to point out that the phenomenon of the pseudo-NG bosons was first considered in S. Weinberg, Phys. Rev. Lett. **29** (1972) 1698; Phys. Rev. D **7** (1973) 2887. As in those papers, the existence of the pseudo-NG bosons in two flavor dense QCD is connected with the presence of an extended symmetry in the leading order, which is *not* a symmetry in the full theory.
26. T. Schaefer, Phys. Rev. D **62** (2000) 094007.
27. D.H. Rischke, Phys. Rev. D **62** (2000) 034007; *ibid.* D **62** (2000) 054017.